

Exit, sunk costs and the selection of firms[★]

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Summary. This paper aims to identify the cost characteristics of exiting firms whenever firms are playing an infinite horizon supergame with time-invariant cost and demand functions. With more than two firms, the problem of which firms exit is quite similar to a coalition formation one. Solving this coalition formation problem, we obtain that the exiting firms are those with higher average cost functions whenever reentry is costless while, whenever reentry is unprofitable, the exiting firms are those with lower marginal (and possibly average) cost functions. Since reentry costs are typically sunk, our analysis points out that the presence of sunk costs affects not only the size (as it is well known) but also the composition of the industry.

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1 Introduction

In markets where firms differ as to their cost functions is it possible to predict what are the cost characteristics of the firms which stay or which exit? In perfectly competitive markets, one can predict that the firms exiting the market are those with highest average costs. Furthermore, for markets with few competitors, this prediction has been extended by Ghemawat and Nalebuff (1985, 1990) and Fudenberg and Tirole (1986) to the case of declining industries. Indeed, using a war of attrition framework, these authors have shown that the less efficient firm will be the first to exit¹. However it is

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¹ For instance, Ghemawat and Nalebuff (1985) show that in a war of attrition with complete information where firms differ according to their capacities, the biggest firm is the first to exit. Since these authors assume that firms incur only a flow maintenance cost which is proportional to their capacity, the largest one has the highest average cost function.

well known that, with imperfect competition, exit can occur in a wide variety of circumstances [see chapters 8 and 9 in Tirole (1988) and Wilson (1992)]. It is then natural to ask whether the above prediction continues to hold in imperfectly competitive markets where firms are not engaged in a war of attrition.

We consider an infinite horizon supergame with discounting, referred to as the production game, played by a set of N firms. Although our results will hold for an arbitrary number of firms, we shall simplify the exposition by assuming that the production game involves only three firms. At each period, firms play a two-stage game where, at the first stage, firms decide simultaneously to stay in or to stay out of the market and, at the second stage, active firms (those which have decided to stay in the market) play a usual Cournot game while the others produce nothing. The discount factor is common to all firms and sufficiently close to one. Note also that the production game will not belong to the class of the war of attrition game since we shall suppose that the market demand and the cost functions are time invariant.

Firms differ according to their limited production capacity as well as to their cost function. We suppose that firms can be ranked according to both their average and marginal cost functions. These rankings may however differ since differences among firms' fixed costs are allowed. Furthermore, whenever a firm has decided to stay out of the market at some period, it incurs a sunk reentry cost if it decides to stay in at some subsequent period. To simplify the analysis, two cases will be examined, namely, the case where reentry is costless and the one where the reentry cost is so large that no firm will find profitable to reenter. Note immediately that, with costless reentry, the production game is simply an infinitely repeated game while, with unprofitable reentry, it loses this repeated game structure (the decision to stay in or to stay out of the market at date t will affect the instantaneous profit function at subsequent periods).

Exit can obviously occur in our framework. We say that a cartel, i.e. a subset of the set of firms, is feasible whenever there exists an equilibrium of the production game where only the members of that cartel stay in the market along the equilibrium path. For the sake of generality we shall suppose that the grand cartel N is feasible. We shall also require that the market cannot be monopolized, that is, no one-firm cartel is feasible. On the other hand, it will be easy to verify that, for a discount factor close to one, the two-firm cartel $\{i, j\}$ is feasible if and only if firm k 's cost function is such that firm k 's minimax payoff in the Cournot game with all firms being active, $v_k(N)$, is strictly negative. Accordingly, many two-firm cartels can be feasible at the same time.

At this point, some important remarks must be made. First, for any demand and firms' cost functions there corresponds a set of feasible cartels. Accordingly, since demand and cost functions are taken as given in our analysis, so is the set of feasible cartels. Second, for a given demand function, a particular set of feasible cartels is compatible with many vectors of firms'

cost functions. For instance, if we know that the set of feasible cartels is given by $\{\{i, j\}, \{i, k\}, N\}$, we only know that $v_i(N)$ is positive while $v_j(N)$ and $v_k(N)$ are both strictly negative. This can arise with firm j having a lower or higher average cost function than firm k 's. Moreover, this is also compatible with firm k having a higher average cost function than firm j 's because it has a higher marginal cost function and lower fixed costs, or a lower marginal cost function and larger fixed costs. Our purpose in this paper is not to determine the set of firms' cost functions which give rise to a particular set of feasible cartels. Our aim will instead be to answer questions like, for instance, if $\{\{i, j\}, \{i, k\}, N\}$ is the set of feasible cartels and if firm j has a lower average cost function than firm k , do we have any reason to think that firm j will be more likely to stay in the market or, equivalently, do we have any reason to think that cartel $\{i, j\}$ is the most likely to form? Remark finally that such kind of questions is of interest only when there are many feasible two-firm cartels and cannot therefore be answered with the help of the production game alone. However, a careful analysis of the production game will be required to characterize the set of payoff vectors which are attainable for each feasible cartel² S , hereafter denoted by $\mathcal{V}_\alpha(S)$.

Hence, to obtain a prediction about the cost characteristics of the exiting firm, we must determine the cost characteristics of the feasible cartel which is the most likely to form. Problems of coalition formation have been extensively analyzed in a wide literature (see Greenberg (1994) for a survey) where a variety of methodologies and concepts are proposed. We adopt here an approach consisting of two steps. First, we shall associate to the production game a game in coalitional form, denoted by (N, V) . The characteristic function V will associate to each coalition or cartel S a set of payoff vectors $V(S)$ as follows: For any feasible two-firm cartel S , $V(S)$ is simply equal to the set of attainable payoff vectors for cartel S , $\mathcal{V}_\alpha(S)$; for the grand cartel N , $V(N)$ will be equal to the set of all equilibrium payoff vectors in the production game; and, for any cartel S which is not feasible, we take the convention that $V(S)$ restricts only the payoff of the members of S to be equal to zero. We then use the core of this coalitional form game to define a stable cartel: Cartel S is said stable if it is feasible and the core of (N, V) has a non-empty intersection with the set of attainable payoff vectors for that cartel, $\mathcal{V}_\alpha(S)$. Accordingly, a feasible cartel S is stable whenever there exists at least one attainable payoff vector for that cartel which cannot be blocked by any other cartel. This means intuitively that the members of a stable cartel can obtain, by staying in the market together, a payoff vector at least as large as any other payoff vector they can obtain if the members of another feasible cartel stay in the market. Hence, stable cartels are the most likely to form.

Note that, starting with the coalition form game (N, V) , a non-cooperative approach to the formation of coalition could have been used. This ap-

² To be quite precise, the set of attainable payoff vectors for cartel S is simply the set of all payoff vectors that firms can obtain at (subgame perfect Nash) equilibrium of the production game where, along the equilibrium path, only the firms in cartel S stay in the market.

proach consists to define an extensive form negotiation game in the spirit of the alternating offers model of Rubinstein (1982) and to analyze the subgame perfect equilibria of this game. Fortunately, a strong relationship between this non-cooperative approach and the one adopted in this paper has been recently established in Moldovanu and Winter (1995)'s work by using the concept of an Order Independent Equilibrium (OIE), that is, a strategy profile such that, for any specification of the first movers in the sequential game, it remains an equilibrium and it leads to the same payoffs. Starting with a game in coalitional form, these authors have indeed provided a quite appealing negotiation game for which (i) payoff vectors resulting from OIE that uses pure stationary strategies belong to the core of the coalitional form game and, (ii) for any payoff vector in the core of the coalitional form game there exists an OIE that uses pure stationary strategies that leads to that payoff vector³. Accordingly, a feasible cartel will be stable if and only if it forms at an OIE (that uses pure stationary strategies) of the Moldovanu and Winter (1995)'s sequential game. This provides another motivation for our definition of a stable cartel.

Concerning the three firms case, our main results are: (i) whenever reentry is costless, if the two-firm cartel containing the firms with lower average cost functions is feasible then either this cartel is the unique stable cartel or no stable cartel exists; (ii) whenever reentry is unprofitable, if the two-firm cartel containing the firms with higher marginal cost functions is feasible then either this cartel is the unique two-firm cartel which is stable or no stable cartel exists; (iii) a sufficient condition for a stable cartel to exist is that there exists a firm which belongs to all feasible cartels. To illustrate these conclusions, let us suppose, on the one hand, that the set of feasible cartels is given by $\{\{i, j\}, \{i, k\}, N\}$ and, on the other hand, that firm j 's average and marginal cost functions are lower than firm k 's. According to the above results, $\{i, j\}$ is the unique stable cartel whenever reentry is costless or, in other words, the firm with the highest average cost function, firm k , is the most likely to exit the market. In this case, we find back the prediction obtained in perfectly competitive markets as well as in declining industries. However, whenever reentry is unprofitable, our results state that $\{i, k\}$ is the unique two-firm cartel which is stable⁴. This means that, whenever reentry is unprofitable, the firm with a lower marginal and average cost function, firm j , will be the more likely to exit the market. This sharply contrasts with the prediction obtained for the costless reentry case.

³ Notice that these results hold if the coalitional form game is superadditive and if every subgame of the coalitional form game has a non-empty core. Our coalitional game is clearly superadditive. Furthermore, with three firms, the second requirement will obviously be satisfied whenever the core of (N, V) is non-empty. For the case of an arbitrary number of firms, a sufficient condition will be given which shall ensure that the core of (N, V) and of its subgames is non-empty.

⁴ In the case of unprofitable reentry, we cannot generally assert that the grand cartel N is not stable. This indicates that the possibility of predation does not imply by itself that predation will effectively occur.

These conclusions will be generalized to the case of an arbitrary number of firms. Note furthermore that reentry costs are sunk. Hence, our analysis points out a novel implication of sunk costs: not only, as it is already well known from the literature on entry preemption, they can determine the number of firms in an industry, but they also enter the determination of the type of firms that stay in a market i.e. the *composition* of the industry.

The plan of the paper is as follows. We first analyze the case with three firms. In Section 2 we introduce our assumptions relative to the cost and demand functions, and we analyse the equilibrium outcomes of the production game. In Section 3 we define the game in coalitional form and we characterize stable cartels for the case of costless reentry and for the case of unprofitable reentry. The results for the three firms case are extended to the case with an arbitrary number of firms in Section 4. Section 5 concludes.

2 The production game

We consider a supgame, denoted by Γ_α , involving a set N of firms. Note immediately that, except in Section 4 where results for an arbitrary number of firms are provided, we restrict the analysis to the case of three firms. Γ_α consists of the infinite repetition of the two-stage game where (i) at the first stage each firm decides to stay in or to stay out of the market, (ii) at the second stage the firms which have decided to stay in the market, hereafter referred to as the active firms, play a usual Cournot game whilst an inactive firm produces nothing. At each stage decisions are made simultaneously; and decisions to stay in or to stay out taken at the first stage are perfectly observed by all firms before they choose their production at the second stage. The scalar α , in the open interval $(0, 1)$, denotes the discount factor common to all firms. Note also that, for all $i \in N$, firm i 's payoff in the production game resulting from the play of a particular strategy profile is defined as the average discounted sum of the sequence of firm i 's payoffs in the two-stage game. Precisely, if a strategy profile leads to a sequence $(w_{it})_{t=0}^\infty$ of firm i 's payoffs in the two-stage game then firm i 's payoff, P_i , in the production game resulting from the play of this strategy profile is given by $(1 - \alpha) \sum_{t=0}^\infty \alpha^t w_{it}$.

The purpose of this preparatory section is twofold. On the one hand, we give a precise content to the concept of a feasible cartel and relate the feasibility of a cartel with the cost and demand functions. On the other hand, for each feasible cartel S , we characterize the set of all payoff vectors that firms can obtain at subgame perfect Nash equilibria of Γ_α where along the equilibrium path only the firms in cartel S stay in the market at each period. This set is referred to as the set of attainable payoff vectors for cartel S .

We shall consider two cases: the case where reentry is costless and the one where reentry is always unprofitable. Clearly, when reentry is costless Γ_α is a repeated game, while Γ_α looses this repeated game structure whenever reentry is unprofitable (the decision to stay in or to stay out of the market at period t will affect the profit that a firm can obtain at period $t + 1$). To analyze these two cases in a unified way we shall introduce auxiliary games. Each of

these games, denoted by $G_\alpha(S)$, involves a set S of firms with common discount factor α , and consists of the infinite repetition of a usual Cournot game. Note that, even if $G_\alpha(N)$ involves the same set of firms than the production game Γ_α , these two games differ. Indeed, contrarily to the production game Γ_α , $G_\alpha(N)$ does not allow firms to decide to stay in or to stay out of the market so that firms' exit is ruled out in $G_\alpha(N)$. We shall therefore refer to $G_\alpha(S)$ as the no-exit game with a set S of firms. Obviously, whether reentry is costless or unprofitable, there will be a strong relationship between the set of subgame perfect equilibria of $G_\alpha(S)$ and the set of subgame perfect equilibria of Γ_α where along the equilibrium path only the firms in S stay in the market.

Using these auxiliary games we shall easily obtain a characterization of the set of attainable payoff vectors for each feasible cartel S (see subsection 2.3 below for the case of costless reentry and subsection 2.4 for unprofitable reentry), and relate the feasibility of a cartel to the cost functions (see subsection 2.2). However, before doing this, we introduce our basic assumptions on the cost and demand functions.

2.1 Assumptions

An active firm has to pay a (time-invariant) fixed cost F_i as well as a variable cost given by the function $c(q_i; \theta_i, \bar{q}_i)$ where θ_i and \bar{q}_i are (time-invariant) firm-specific parameters and q_i stands for quantity⁵. If a firm decides to stay out, it produces nothing and incurs no cost. Furthermore if a firm, say i , has decided to stay out at period $t - 1$, it must pay a reentry cost, R_i , if it decides to stay in the market at period t . We simplify the analysis by considering in turn two polar cases, namely, the case where, for all firms, the reentry cost is so large that reentry is always unprofitable and the one where reentry is costless.

The variable cost function c depends upon two firm-specific parameters, θ_i and \bar{q}_i . The parameter \bar{q}_i stands for the firm i 's capacity constraint and is thus strictly positive. Accordingly c is only defined for $0 \leq q_i \leq \bar{q}_i$. On the other hand, θ_i is introduced to rank firms according to their marginal cost function. We shall indeed suppose that for any quantity q such that the marginal cost to produce this quantity is well defined for firms i and j , firm j 's marginal cost is strictly greater than the firm i 's if and only if $\theta_j > \theta_i$. More precisely, let $X_i = [0, \bar{q}_i]$, we require the following.

Assumption 1 *The variable cost function is twice continuously differentiable with respect to q_i and θ_i on $X_i \times R_{++}$ and satisfies (a) $c(0; \theta_i, \bar{q}_i) = 0$, $\forall \theta_i \geq 0$; (b) $\forall (\theta_i, q_i) \in R_+ \times X_i$, $0 \leq \partial c(q_i; \theta_i, \bar{q}_i) / \partial q_i \leq \eta_i < \infty$; and (c) $\forall (\theta_i, q_i) \in R_+ \times X_i$, $\partial^2 c(q_i; \theta_i, \bar{q}_i) / \partial q_i \partial \theta_i > 0$.*

⁵ The time index that should be assigned to the quantity variable is omitted as long as this is unnecessary.

The average cost function of firm i is denoted by $\mu_i(q)$, i.e. $\mu_i(q) = [c(q; \theta_i, \bar{q}_i) + F_i]/q$. To simplify the analysis, we assume that firms can also be ranked according to their average cost function.

Assumption 2 For any $i, j \in N$, either $\mu_i(q) > \mu_j(q)$ for all $q \in X_i \cap X_j$, or $\mu_i(q) < \mu_j(q)$ for all $q \in X_i \cap X_j$, or $\mu_i(q) = \mu_j(q)$ for all $q \in X_i \cap X_j$.

Now let Q stand for aggregate output. At each period, the inverse demand function for the homogeneous good, denoted $f(Q)$, satisfies:

Assumption 3 For all $Q \in [0, \sum_{i \in N} \bar{q}_i]$, f is twice continuously differentiable with $f(Q) \geq 0$ and $\partial f / \partial Q < 0$, for all Q such that $f(Q) > 0$.

For an active firm the profit function in the Cournot game will be written as:

$$\pi_i(q_i, Q_{-i}) = f(q_i + Q_{-i})q_i - c(q_i; \theta_i, \bar{q}_i) - F_i$$

where $Q_{-i} = Q - q_i$. Furthermore, for a given set S of active firms, the set of feasible payoff vectors in the Cournot game will be denoted by $\mathcal{A}(S)$. Precisely, for any $S \subseteq N$, let $X^S = \times_{i \in S} X_i$, we have:

$$\mathcal{A}(S) = \{(\pi_i)_{i \in S} | \exists q \in X^S \text{ such that } \pi_i = \pi_i(q_i, Q_{-i}) \forall i \in S\}$$

We shall assume:

Assumption 4 (a) For all $i \in N$, π_i is strictly quasi-concave in (q_i, Q_{-i}) on $X_i \times [0, \sum_{j \neq i} \bar{q}_j]$; (b) $\partial \pi_i(\bar{q}_i, Q_{-i}) / \partial q_i \leq 0$ for all $Q_{-i} \in [\min\{\bar{q}_j | j \in N \text{ and } j \neq i\}, \sum_{j \neq i} \bar{q}_j]$, for all $i \in N$; and (c) $\mathcal{A}(N) \cap R_{++}^{|N|} \neq \emptyset$ and $\mathcal{A}(S)$ is convex for any $S \subseteq N$.

Obviously assumption 4-(a), together with the restriction that any active firm i must choose a quantity in $[0, \bar{q}_i]$ and our differentiability assumptions, ensures that a Cournot equilibrium exists. Furthermore, assumption 4-(b) is a convenient way to impose that capacities are such that at least one firm is unconstrained at a Cournot equilibrium and this holds even if only two firms remain active on the market. Note also that, in the first part of assumption 4-(c), we require the existence of a strictly positive feasible payoff vector whenever three firms are active. This could be assumed away but it is kept for the sake of generality since, as we shall see below, this will ensure that there always exist some equilibria in the production game such that all firms stay in along the equilibrium path. Finally, the convexity assumption of $\mathcal{A}(S)$ is introduced for simplicity. It imposes mild restrictions on the demand and cost functions⁶.

⁶ Suppose that $S = \{i, j\}$ and let $\Pi_i(\bar{\pi}_j)$ be equal to $\max_{q_i \in X_i, q_j \in X_j} \pi_i(q_i, q_j)$ subject to $\pi_j(q_j, q_i) = \bar{\pi}_j$. Then $\mathcal{A}(S)$ will be convex if and only if $\Pi_i(\bar{\pi}_j)$ is concave. Now, from concave programming (see Theorem 4.7 in Beavis and Dobbs, for instance), we know that the concavity of π_i and π_j in (q_i, q_j) is sufficient to ensure the concavity of Π_i . Accordingly, any restrictions on the demand and cost function which ensure that π_i is concave in (q_i, q_j) are sufficient for $\mathcal{A}(S)$ to be convex. Note however that the convexity of $\mathcal{A}(S)$ does not prevent firms' marginal costs to be constant even if the inverse demand function is linear (see Friedman, 1987, p. 25, for an example).

Now let us denote by $v_i(S)$ the minimax payoff of firm i in the Cournot game with a set S of active firms. Since, under assumption 3, π_i is a decreasing function of Q_{-i} we have

$$v_i(S) = \pi_i \left(q_i^R \left(\sum_{j \in S \setminus i} \bar{q}_j \right), \sum_{j \in S \setminus i} \bar{q}_j \right)$$

where $q_i^R(Q_{-i}) = \arg \max_{q_i \in X_i} \pi_i(q_i, Q_{-i})$. We shall require:

Assumption 5 For any $S \subset N$ such that $|S| = 2$, $v_i(S) > 0 \forall i \in S$.

As we shall see, this assumption will guarantee that the market cannot be monopolized. It must be noted that assumption 5 requires that any firm's capacity is bounded above and that this upper bound is not "too large". Indeed, if it were not the case then $v_i(S)$ would be equal to $-F_i$ for any S such that $|S| \geq 2$ and assumption 5 would be violated.

2.2 Feasible cartels

At the outset, let us define what we call a feasible cartel:

Definition 1 A cartel $S \subseteq N$ is said feasible if there exists a subgame perfect equilibrium σ of the production game Γ_α such that, along the equilibrium path generated by σ , firms in S stay in and firms in $N \setminus S$ stay out of the market.

We shall denote by \mathcal{F} the set of feasible cartels. The purpose of this subsection is to make precise the relationship between the set of feasible cartels and the cost and demand functions. In order to do this in a simple way, we shall use the no-exit games defined at the beginning of this Section. But, before we proceed, it must be immediately noted that an appropriate version of the Folk Theorem (see Wen, 1994) can be applied to characterize the set of equilibrium (discounted average) payoffs⁷ of the no-exit games. To be precise, let $\mathcal{G}_\alpha(S)$ denote the set of equilibrium payoff vectors of the no-exit game with a set S of firms, $G_\alpha(S)$. Let furthermore $\mathcal{G}(S)$ denote the set of feasible and strictly individually rational payoff vectors of the Cournot game where the set of active firms is given by S , that is,

$$\mathcal{G}(S) = \{(P_i)_{i \in S} \in \mathcal{A}(S) | P_i > v_i(S) \forall i \in S\}$$

Then we have from the Folk Theorem that for any $P^0 \in \mathcal{G}(S)$ there exists $\alpha^0 < 1$ such that $P^0 \in \mathcal{G}_\alpha(S)$ for any $\alpha \in (\alpha^0, 1)$.

Whenever α tends to 1, an assumption we shall make throughout the paper, N is a feasible cartel. Indeed, from assumption 4-(c), there exists a payoff vector in $\mathcal{G}_\alpha(N)$ which gives to all firms a strictly positive payoff, that is, there exists a subgame perfect equilibrium of $G_\alpha(N)$, $\tilde{\sigma}$, leading to a strictly positive payoff for all firms. Obviously, $\tilde{\sigma}$ can be used to construct a subgame

⁷ Given a path $(q_t)_{t=0}^\infty$ in $G_\alpha(S)$, with $q_t \in X^S$, the resulting discounted average payoff of firm i is $(1 - \alpha) \sum_{t=0}^\infty \alpha^t \pi_i(q_{it}, Q_{-it})$.

perfect equilibrium σ of the production game Γ_α where all firms stay in the market at each period along the equilibrium path generated by σ . N is therefore feasible and \mathcal{F} is thus always non-empty. On the other hand, note that assumption 5 ensures that no one-firm cartel belongs to \mathcal{F} i.e. that the market cannot be monopolized⁸.

We then turn to the condition under which a two-firm cartel is feasible. A necessary condition for cartel $\{i, j\}$ to be feasible is that the minimax payoff of the third firm, say k , in the Cournot game with the three firms being active, $v_k(N)$, is strictly negative. Indeed, if $v_k(N)$ is non-negative then firm k will obtain a strictly positive payoff at all equilibria of the no-exit game $G_\alpha(N)$. It will therefore be impossible to construct an equilibrium in the production game Γ_α where firm k stays out of the market along the equilibrium path. On the other hand, whenever α tends to 1, the condition $v_k(N) < 0$ is also sufficient for $\{i, j\}$ to be feasible. Indeed, the minimal payoff firm k can obtain in $\mathcal{G}_\alpha(N)$ being strictly negative since it tends to $v_k(N)$ as α tends to 1, it will therefore be possible to construct an equilibrium of the production game Γ_α where firm k stays out of the market along the equilibrium path. Consequently, whenever α tends to 1, $\{i, j\}$ is feasible if and only if $v_k(N) < 0$. This implies that the feasibility of a two-firm cartel depends only on the demand and cost functions.

For a given demand function and a given capacity for firms i and j , many firm k 's cost functions will imply that $v_k(N)$ is strictly negative: some of them will be characterized by a large fixed cost, F_k , and others will display a large cost parameter θ_k ⁹. Remark also that, due to the cost differences between firms, a strictly negative $v_k(N)$ does not prevent $v_j(N)$, for instance, to be strictly negative. In other words, the feasibility of $\{i, j\}$ does not exclude the feasibility of cartel $\{i, k\}$. Now, since the demand and cost functions are *a priori* given so is the set of feasible cartels. Our analysis will therefore take the set of feasible cartels as exogenously given and will aim to determine the feasible cartel which will be the more likely to form i.e. the set of firms remaining on the market.

2.3 Equilibrium payoffs with costless reentry

We now characterize, for each feasible cartel S , the set of all payoff vectors that firms can obtain at subgame perfect equilibria of the production game Γ_α where, along the equilibrium path, only the firms in cartel S stay in the market. We refer to this set as the set of attainable payoff vectors for cartel S and denote it by $\mathcal{V}_\alpha(S)$.

⁸ Indeed, let σ^0 be a strategy profile in Γ_α which specifies that firms j and k stay out of the market at some period t^0 . But, if firm j deviates at t^0 by deciding to stay in and to produce a quantity given by its best-reply $q_j^R(q_i)$, it will increase its payoff by at least $v_j(i, j)$. Then, since $v_j(i, j)$ is strictly positive by assumption 5, σ^0 cannot be a subgame perfect equilibrium of Γ_α .

⁹ For instance, if the demand function writes as $D - Q$ and firm k 's cost function is given by $\theta_k q_k + F_k$, then $\{i, j\}$ is feasible whenever (θ_k, F_k) is such that $\theta_k + 2\sqrt{F_k} > D - (\bar{q}_i + \bar{q}_j)$.

To begin with, let us consider cartel N . It is quite easy to verify that, whenever α tends to 1, any subgame perfect equilibrium σ of the no-exit game $G_\alpha(N)$ which gives to any firm i a payoff strictly greater than $\max\{0, v_i(N)\}$ ¹⁰ can be used to construct a subgame perfect equilibrium $\tilde{\sigma}$ of the production game Γ_α which leads to the same payoff vector. Therefore, using the Folk Theorem, we have that, for any payoff vector P^0 which gives to each firm i a payoff strictly greater than $\max\{0, v_i(N)\}$ and which is feasible, i.e. $P^0 \in \mathcal{G}(N) \cap R_{++}^{[N]}$, there exists $\alpha^0 < 1$ such that, for all $\alpha \in (\alpha^0, 1)$, P^0 belongs to $\mathcal{V}_\alpha(N)$. For further references, we shall denote by $v_{zi}(N)$ the minimal payoff that firm i can obtain in $\mathcal{V}_\alpha(N)$. Note also that the set of attainable payoff vectors for cartel N does not depend on the assumption on the possibility of reentry.

Consider next a feasible two-firm cartel. We claim that

Claim 1 *Let S be a feasible two-firm cartel and let reentry be costless. Then,*

1. $\forall \alpha \in (0, 1)$, $\mathcal{V}_\alpha(S) \subseteq \{(P_i)_{i \in N} | (P_i)_{i \in S} \in \mathcal{A}(S), P_i \geq \max\{0, v_i(N)\} \forall i \in S, P_j = 0 \forall j \in N \setminus S\}$,
2. $\forall P^0 \in \{(P_i)_{i \in N} | (P_i)_{i \in S} \in \mathcal{A}(S), P_i > \max\{0, v_i(N)\} \forall i \in S, P_j = 0 \forall j \in N \setminus S\}$, *there exists $\alpha^0 < 1$ such that, for all $\alpha \in (\alpha^0, 1)$, $P^0 \in \mathcal{V}_\alpha(S)$.*

Let the minimal payoff that any member, say i , of a feasible cartel S can obtain in $\mathcal{V}_\alpha(S)$ be denoted by $v_{zi}(S)$. Then the point in Claim 1 to be emphasized is that $v_{zi}(S)$ tends to $\max\{0, v_i(N)\}$ if α tends to 1. To see why, remark first that since reentry is costless any subgame starting at period $t + 1$ is not affected by the decision of a firm to stay out of the market at period t . In other words, any subgame starting at period $t + 1$ coincides with the production game Γ_α except that the payoffs must be appropriately discounted. This implies that, any firm i 's deviation at period t can be punished by the play, in the subgame starting at period $t + 1$, of an equilibrium in the production game Γ_α which gives to firm i a (discounted average) payoff equal to $v_{zi}(N)$. Therefore, if firm i deviates at period 0, it will obtain an average discounted payoff, P_i^d , equal to $(1 - \alpha)d_i + \alpha v_{zi}(N)$, where d_i stands for the firm i 's instantaneous profit from the deviation at period 0. Now, whenever α tends to 1, we have seen that $v_{zi}(N)$ tends to $\max\{0, v_i(N)\}$ and therefore P_i^d tends also to $\max\{0, v_i(N)\}$. Hence, any strategy profile in the production game Γ_α leading to a firm i 's payoff smaller than $\max\{0, v_i(N)\}$ cannot be a subgame perfect equilibrium of the production game for any $\alpha < 1$. Consequently, for all $i \in S$, $v_{zi}(S)$ cannot be smaller than $\max\{0, v_i(N)\}$ for any $\alpha < 1$.

Let us now compare the payoffs that firms can obtain in the set of attainable payoffs for different feasible cartels. To begin with, consider first the

¹⁰ Notice that, when $v_i(N)$ is strictly negative, there exist equilibria of $G_\alpha(N)$ which give to firm i a strictly negative payoff. This comes from the fact that exit is ruled out in $G_\alpha(N)$. It is however obvious that such equilibria cannot be used to construct equilibria of the production game leading to the same payoff.

payoffs that firms i and j can obtain in $\mathcal{V}_\alpha(i, j)$ and in $\mathcal{V}_\alpha(N)$. Remark immediately that, at any equilibrium of the production game leading to a payoff vector in $\mathcal{V}_\alpha(i, j)$, firm k stays out of the market at each period and does not produce anything, while, at any equilibrium leading to a payoff vector in $\mathcal{V}_\alpha(N)$, firm k will produce a strictly positive quantity. This implies that firms i and j can obtain a larger payoff in $\mathcal{V}_\alpha(i, j)$ than in $\mathcal{V}_\alpha(N)$. Moreover, the minimal payoff that firm i , resp. j , can obtain in $\mathcal{V}_\alpha(i, j)$ as well as in $\mathcal{V}_\alpha(N)$ tends to $\max\{0, v_i(N)\}$, resp. $\max\{0, v_j(N)\}$. Consequently, denoting by $\partial\mathcal{V}_\alpha(i, j)$ the set of undominated payoff vectors for firm i and j in $\mathcal{V}_\alpha(i, j)$ i.e. $\partial\mathcal{V}_\alpha(i, j) = \{P \in \mathcal{V}_\alpha(i, j) \mid \nexists P' \in \mathcal{V}_\alpha(i, j) \text{ such that } P'_i > P_i \text{ and } P'_j > P_j\}$, we have

Lemma 1 *Let S be a feasible two-firm cartel and let reentry be costless. Then, for any $P \in \mathcal{V}_\alpha(N)$ there exists $P' \in \mathcal{V}_\alpha(S)$ such that $P'_i > P_i$ for all $i \in S$ and, conversely, for any $P \in \partial\mathcal{V}_\alpha(S)$ there does not exist $P' \in \mathcal{V}_\alpha(N)$ such that $P'_i > P_i$ for all $i \in N$.*

Consider next the payoffs that firms can obtain in $\mathcal{V}_\alpha(i, j)$ and $\mathcal{V}_\alpha(i, k)$. To begin with, let us denote by $\Pi_i^*(\bar{\pi}, j)$ the maximal payoff that firm i can obtain in the set of feasible payoff vectors $\mathcal{A}(i, j)$ when firm j 's payoff is given by $\bar{\pi}$, that is,

$$\Pi_i^*(\bar{\pi}, j) = \max_{q_i \in X_i, q_j \in X_j} \{\pi_i(q_i, q_j) \text{ subject to } \pi_j(q_j, q_i) = \bar{\pi}\}$$

We define $\Pi_i^*(\bar{\pi}, k)$ in the same way. Let us furthermore denote by $\hat{q}_j(q_i, \bar{\pi})$ the minimal quantity that firm j must produce to obtain a payoff equal to $\bar{\pi}$ when firm i produces q_i , that is, $\hat{q}_j(q_i, \bar{\pi}) = \min\{q_j \mid \pi_j(q_j, q_i) = \bar{\pi}\}$. Clearly, the smaller $\hat{q}_j(q_i, \bar{\pi})$ is, the larger $\Pi_i^*(\bar{\pi}, j)$ is. Now, defining $\hat{q}_k(q_i, \bar{\pi})$ as we do for firm j , it is easy to see that $\hat{q}_k(q_i, \bar{\pi}) > \hat{q}_j(q_i, \bar{\pi})$ whenever $\theta_j = \theta_k$ and $F_j < F_k$. More generally, if firm j has a lower average cost function than firm k 's, i.e. $\mu_j < \mu_k$ ¹¹, then $\hat{q}_k(q_i, \bar{\pi}) > \hat{q}_j(q_i, \bar{\pi})$ and consequently $\Pi_i^*(\bar{\pi}, k) < \Pi_i^*(\bar{\pi}, j)$. This is what we call the *efficiency effect*: *Provided firm i 's partner in both two-firm cartels receives the same payoff $\bar{\pi}$, firm i can obtain a larger payoff by associating with the firm having the lower average cost function.*

Remark now that, since $\{i, j\}$ and $\{i, k\}$ are supposed feasible, $v_j(N)$ and $v_k(N)$ are both strictly negative and the minimal payoff that firm j , resp. firm k , can obtain in $\mathcal{V}_\alpha(i, j)$, resp. $\mathcal{V}_\alpha(i, k)$, tends to 0 if α tends to 1. This, together with the fact that Π_i^* is a decreasing function of $\bar{\pi}$, implies that the maximal payoff that firm i can obtain in $\mathcal{V}_\alpha(i, j)$, $v_{\alpha i}^M(i, j)$, tends to $\Pi_i^*(0, j)$ when α tends to 1. Similarly, $v_{\alpha i}^M(i, k)$ tends to $\Pi_i^*(0, k)$ when α tends to 1. We therefore have

¹¹ Let $f : F \rightarrow R$ and $g : G \rightarrow R$ be real-valued functions. Then, we use the notation $f > g$ to denote that $f(x) > g(x)$ for any $x \in F \cap G$ and similarly, $f < g$ stands for $f(x) < g(x)$ for any $x \in F \cap G$.

Lemma 2 *Let $\{i, j\}$ and $\{i, k\}$ be feasible and let reentry be costless, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if $\mu_j < \mu_k$ then (a) $v_{zi}^M(i, j) > v_{zi}^M(i, k)$ and (b) $\forall P \in \mathcal{V}_\alpha(i, k)$ there exists $P' \in \mathcal{V}_\alpha(i, j)$ such that $P'_i > P_i$ and $P'_j > P_j$.*

The second part of the Lemma follows immediately from the first one since, at any payoff vector in $\mathcal{V}_\alpha(i, k)$, firm j obtains a zero payoff while it receives a strictly positive payoff at any payoff vector in $\mathcal{V}_\alpha(i, j)$

2.4 Equilibrium payoffs with unprofitable reentry

Since, as noted previously, the set of attainable payoff vectors for cartel N does not depend on the assumption about the possibility of reentry, the first purpose of this subsection is to characterize the set of attainable payoff vectors for a feasible two-firm cartel. We claim:

Claim 2 *Let S be a feasible two-firm cartel and let reentry be unprofitable. Then,*

1. $\forall \alpha \in (0, 1)$, $\mathcal{V}_\alpha(S) \subseteq \{P | (P_i)_{i \in S} \in \mathcal{A}(S), P_i \geq v_i(S) \forall i \in S \text{ and } P_k = 0 \forall k \in N \setminus S\}$,
2. $\forall P^0 \in \{P | (P_i)_{i \in S} \in \mathcal{A}(S), P_i > v_i(S) \forall i \in S \text{ and } P_k = 0 \forall k \in N \setminus S\}$, there exists $\alpha^0 < 1$ such that, for all $\alpha \in (\alpha^0, 1)$, $P^0 \in \mathcal{V}_\alpha(S)$.

These two results are obtained by using the no-exit game with a set S of firms, $G_\alpha(S)$, which consists of the infinite repetition of the Cournot game with a set S of active firms. For the ease of exposition, let us consider that cartel $\{i, j\}$ is feasible. Then, by assumption 5, we have that the minimal payoff that firms i and j can obtain in the Cournot game with a set $\{i, j\}$ of active firms, i.e. $v_i(i, j)$ and $v_j(i, j)$, is strictly positive. This implies that firms i and j will obtain a strictly positive payoff at any equilibrium of the no-exit game $G_\alpha(i, j)$. Therefore, any equilibrium σ^0 of the no-exit game $G_\alpha(i, j)$ can be used to construct an equilibrium $\tilde{\sigma}$ of the production game Γ_α where along the equilibrium path generated by $\tilde{\sigma}$ only firms i and j stay in the market and which leads to the same payoff for firms i and j than the one they obtain at¹² σ^0 . Then, since the no-exit game $G_\alpha(i, j)$ is a standard repeated game, we obtain the second part of Claim 2 by using the Folk Theorem.

The first part of Claim 2 states that the minimal payoff that firm i , for instance, can obtain in $\mathcal{V}_\alpha(i, j)$ cannot be smaller than $v_i(i, j)$, that is, firm i 's minimax payoff in the Cournot game with a set $\{i, j\}$ of active firms. To see this, let us first consider a subgame of the production game Γ_α which starts at period 1 and follows the decisions to stay in by firms i and j and to stay out by firm k . We denote such a subgame by $\Gamma_\alpha^{\{i, j\}}$ and we let, for convenience, the payoff in a subgame $\Gamma_\alpha^{\{i, j\}}$ be discounted back to period 1. Two remarks

¹² In other words, any payoff vector P^0 such that P_k^0 equals zero and (P_i^0, P_j^0) belongs to the set of equilibrium payoff vectors of the no-exit game $G_\alpha(i, j)$, belongs to the set of attainable payoff vectors for cartel $\{i, j\}$, $\mathcal{V}_\alpha(i, j)$.

must be immediately made about the equilibria of such a subgame. First, since reentry is unprofitable, a necessary condition for a strategy profile $(\sigma_i, \sigma_j, \sigma_k)$ to be a subgame perfect equilibrium of $\Gamma_\alpha^{\{i,j\}}$ is that firm k 's strategy σ_k specifies that, at each period $t \geq 1$ and *whatever the history at period t* firm k stays out of the market and produces nothing¹³. Second, since the minimal (instantaneous) profit that firms i and j can obtain if they stay in, $v_i(i, j)$ and $v_j(i, j)$, is strictly positive by assumption 5, a strategy profile $(\sigma_i, \sigma_j, \sigma_k)$ will be a subgame perfect equilibrium of $\Gamma_\alpha^{\{i,j\}}$ only if σ_i and σ_j specify that, at each period $t \geq 1$ and *whatever the history at period t* , firms i and j stay in the market. These two remarks lead thus to the conclusion that any equilibrium σ^0 of a subgame $\Gamma_\alpha^{\{i,j\}}$ can be used to construct an equilibrium $\tilde{\sigma}$ of the no-exit game $G_\alpha(i, j)$ leading to the same payoff for firms i and j than the one they obtain at σ^0 . As the converse is also true, we obtain that the set of equilibrium payoffs that firms i and j can obtain in a subgame $\Gamma_\alpha^{\{i,j\}}$ coincides with the set of equilibrium payoff vectors of the no-exit game $G_\alpha(i, j)$. This means in particular that the minimal payoff that firm i , for instance, can obtain at an equilibrium of a subgame $\Gamma_\alpha^{\{i,j\}}$ is equal to the minimal payoff that firm i can obtain at an equilibrium of $G_\alpha(i, j)$. We denote the latter by $g_{xi}(i, j)$. Note that $g_{xi}(i, j)$ is greater than $v_i(i, j)$ for all α and, by the Folk Theorem, $g_{xi}(i, j)$ tends to $v_i(i, j)$ as α tends to 1.

Now, if firm i deviates at period 0 from the quantity specified by an equilibrium in the production game Γ_α it can be punished by the play, in the subgame starting at period 1, of an equilibrium which gives to firm i a payoff equal to $g_{xi}(i, j)$. Therefore, the minimal payoff that firm i can obtain by such a deviation, P_i^d , is equal to $(1 - \alpha)d_i + \alpha g_{xi}(i, j)$, where d_i stands for firm i 's instantaneous profit from the deviation at period 0. Hence, since d_i and $g_{xi}(i, j)$ are both greater than $v_i(i, j)$, P_i^d is greater than $v_i(i, j)$ and the first part of Claim 2 follows.

The main result provided by Claim 2 is that, with unprofitable reentry, the minimal payoff that a member, say firm j , of a feasible cartel, say $\{i, j\}$, can obtain in the set of attainable payoff vectors for that cartel, $\mathcal{V}_\alpha(i, j)$, tends to its minimax payoff in the Cournot game with a set $\{i, j\}$ of active firms, $v_j(i, j)$, as α tends to 1. This implies that, whenever α tends to 1, the maximal payoff that firm i can obtain in $\mathcal{V}_\alpha(i, j)$, $v_{xi}^M(i, j)$, tends to $\Pi_i^*(v_j(i, j), j)$ with Π_i^* defined as in the previous subsection, that is,

$$\Pi_i^*(v_j(i, j), j) = \max_{q_i \in X_i, q_j \in X_j} \{ \pi_i(q_i, q_j) \text{ subject to } \pi_j(q_j, q_i) = v_j(i, j) \}$$

Remark immediately that, since firm j 's fixed costs enter additively in the right-hand side and the left-hand side of the constraint $\pi_j(q_j, q_i) = v_j(i, j)$, $\Pi_i^*(v_j(i, j), j)$ depends only on firm j 's marginal cost parameter θ_j . Conse-

¹³ This contrasts sharply with the costless reentry case, since, with costless reentry, an equilibrium strategy for firm k will generally specify that it reenters on the market along out-of-equilibrium paths. With unprofitable reentry, the reentry cost is assumed to be large enough for firm k to always prefer to obtain a zero payoff by staying out rather than to reenter on the market by paying the reentry cost.

quently, whenever α tends to 1, the comparison between the maximal payoff firm i can obtain in $\mathcal{V}_\alpha(i, j)$ and in $\mathcal{V}_\alpha(i, k)$ (assuming that $\{i, k\}$ is also feasible) will only involve a comparison between firm j 's and firm k 's marginal cost parameter, θ_j and θ_k . This provides a first reason for which the unprofitable reentry case differs from the costless reentry one. Indeed, with costless reentry the difference between firm i 's maximal payoff in $\mathcal{V}_\alpha(i, j)$ and in $\mathcal{V}_\alpha(i, k)$ depends upon the *average* cost function of firms j and k while, with unprofitable reentry, it depends only upon the *marginal* cost function of firms j and k .

The unprofitable reentry case differs however from the costless reentry one in another respect. To see this clearly, let us remark that $\Pi_i^*(v_j(i, j), j) - \Pi_i^*(v_k(i, k), k)$ can be rewritten as follows:

$$\begin{aligned} \Pi_i^*(v_j(i, j), j) - \Pi_i^*(v_k(i, k), k) &= [\Pi_i^*(v_j(i, j), j) - \Pi_i^*(v_j(i, j), k)] \\ &\quad + [\Pi_i^*(v_j(i, j), k) - \Pi_i^*(v_k(i, k), k)] \end{aligned}$$

The first term in the right-hand side of this expression is what we have called the *efficiency effect*. On the other hand, we shall refer to the second term as the *minimax effect*. It comes indeed from the difference between firm j 's minimax payoff $v_j(i, j)$ and firm k 's one, $v_k(i, k)$. Obviously, this effect is negative (positive) whenever $v_j(i, j)$ is strictly greater (smaller) than $v_k(i, k)$. We now show that these two effects works in opposite directions. To do this in an easy way, let us suppose, without loss of generality (see our above discussion), that fixed costs of firms j and k are equal. Then, if firm j has a lower marginal cost function than that of firm k , that is $\theta_j < \theta_k$, then firm j has also a lower average cost function than that of firm k , that is $\mu_j < \mu_k$. Hence, as we have seen in the previous subsection, the efficiency effect will be positive. On the other hand, $\theta_j < \theta_k$ will also imply that $v_j(i, j)$ is strictly greater than $v_k(i, k)$ so that the minimax effect is negative. Consequently, the difference between the maximal payoff that firm i can obtain in $\mathcal{V}_\alpha(i, j)$ and in $\mathcal{V}_\alpha(i, k)$ will be determined by the interaction of two opposite effects when reentry is unprofitable. This contrasts with costless reentry since we have seen that only the efficiency effect matters in this case. The reason is simply that with costless reentry the minimal payoff of firm j and the one of firm k tend both to zero whenever α tends to 1 so that the minimax effect disappears. We furthermore prove in the Appendix that the minimax effect dominates the efficiency effect. Hence we have

Lemma 3 *Let $\{i, j\}$ and $\{i, k\}$ be feasible and let reentry be unprofitable, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, we have: (a) $v_{ai}^M(i, j) < v_{ai}^M(i, k)$ if and only if $\theta_j < \theta_k$; and (b) $\forall P \in \mathcal{V}_\alpha(i, j)$ there exists $P' \in \mathcal{V}_\alpha(i, k)$ such that $P'_i > P_i$ and $P'_k > P_k$, if and only if $\theta_j < \theta_k$.*

Now the minimax effect will also enter in the comparison of the maximal payoff that firm i can obtain in $\mathcal{V}_\alpha(i, j)$, $v_i^M(i, j)$, and in $\mathcal{V}_\alpha(N)$, $v_i^M(N)$. Indeed, whenever α tends to 1, $v_i^M(N)$ tends to $\Pi_i^*(0, 0, N)$ where

$$\begin{aligned}\Pi_i^*(\bar{\pi}_j, \bar{\pi}_k, N) &= \max_{(q_i, q_j, q_k) \in X^N} \{\pi_i(q_i, Q_{-i}) \text{ subject to } \pi_j(q_j, Q_{-j}) \\ &= \bar{\pi}_j \text{ and } \pi_k(q_k, Q_{-k}) = \bar{\pi}_k\}.\end{aligned}$$

Accordingly, we can write

$$\begin{aligned}\Pi_i^*(v_j(i, j), j) - \Pi_i^*(0, 0, N) &= [\Pi_i^*(v_j(i, j), j) - \Pi_i^*(v_j(i, j), 0, N)] \\ &+ [\Pi_i^*(v_j(i, j), 0, N) - \Pi_i^*(0, 0, N)]\end{aligned}$$

The second term in this expression is the minimax effect which is clearly negative since $v_j(i, j)$ is strictly positive by assumption 5. On the other hand, we have seen in the previous subsection that the first term is strictly positive. So, contrarily to what happens with costless reentry, firm i could obtain a greater payoff in $\mathcal{V}_\alpha(N)$ than in $\mathcal{V}_\alpha(i, j)$. However this will arise only for payoff vectors in $\mathcal{V}_\alpha(N)$ for which firm j 's payoff is strictly smaller than $v_j(i, j)$. Hence, we have

Lemma 4 *Let $\{i, j\}$ be feasible and let reentry be unprofitable. Then, for all P belonging to $\partial \mathcal{V}_\alpha(i, j)$ there does not exist P' belonging to $\mathcal{V}_\alpha(N)$ such that $P'_i > P_i$ and $P'_j > P_j$.*

3 Stable cartels

The production game defines only the set of payoffs that are attainable for a feasible cartel. Hence this game cannot give a prediction about the feasible cartel which is likely to form since there are in general many feasible cartels. To obtain such a prediction we are actually confronted with a problem of coalition formation. Obviously, the formation of coalition has been analyzed by a large literature (see Greenberg, 1994, for a survey) where a variety of methodologies and concepts are proposed.

We approach this coalition formation problem in two steps. First, we shall associate to the production game a game in coalitional form (N, V) where V is the characteristic function. Taking into account the analysis of the production game, we shall let $V(S)$, for any $S \neq N$, be equal to either $\mathcal{V}_\alpha(S)$ whenever S is feasible or $\{P \in R_+^N | P_i = 0, \forall i \in S\}$ whenever S does not belong to the set of feasible cartel. This means simply that a cartel which is not feasible can only ensure, on its own, a zero payoff for its members while the members of a feasible cartel S ($\neq N$) can attain on their own any payoff resulting from the play of a subgame perfect equilibrium of Γ_α characterized by the fact that all members of S remain on the market along the path generated by such an equilibrium. We then define $V(N)$ as the set of all equilibrium payoff vectors in the production game, that is, $V(N) = \cup_{S \in \mathcal{F}} \mathcal{V}_\alpha(S)$. Let then $\mathcal{C}(N, V)$ denote the core of (N, V) , that is,

$$\mathcal{C}(N, V) = \{P \in V(N) | \nexists S' \text{ and } P' \in V(S') \text{ such that } P'_i > P_i \forall i \in S'\}$$

We define a stable cartel as follows:

Definition 2 A cartel $S \subseteq N$ is said stable if it is feasible and $\mathcal{V}_\alpha(S)$, its set of attainable payoff vectors, has a non-empty intersection with $\mathcal{C}(N, V)$.

Accordingly, a feasible cartel S is stable whenever there exists at least one payoff vector, P , belonging to the set of attainable payoff vector for cartel S such that there does not exist a feasible cartel S' and an attainable payoff vector for cartel S' , P' , which gives to all members of S' strictly more than they obtain at P i.e. $P'_i > P_i \forall i \in S'$. This means intuitively that the members of a stable cartel can obtain, by staying in the market together, a payoff at least as large as any other payoff they can obtain if the members of another feasible cartel stay in the market. Hence, stable cartels are the more likely to form.

Although this approach is quite appealing and, as we shall see, quite powerful, it is worthwhile to note that another kind of motivation for this definition of a stable cartel can be found in Moldovanu and Winter (1995)'s work. These authors tackle the problem of coalition formation by defining a non-cooperative negotiation game in extensive form [which is a generalization of the alternating offers model of Rubinstein (1982)] to analyze its subgame perfect equilibria. More precisely, they analyze a non-cooperative game of coalition formation based on an underlying game in coalitional form and define an Order Independent Equilibrium (OIE) as a strategy profile such that, for any specification of the first movers in the sequential game, it remains an equilibrium and it leads to the same payoffs. Assuming that the underlying game in coalitional form is superadditive, they show that payoffs resulting from OIE that use pure stationary strategies belong to the core of the coalitional form game, and, if the latter game has the property that every of its subgames has a nonempty core, then for any payoff vector in the core there exists an OIE in pure stationary strategies that leads to that payoff vector. With three firms¹⁴, (N, V) satisfies the requirements for these results to be applied. Consequently, a cartel will be stable if and only if this cartel will form at a non-cooperative OIE (that uses pure stationary strategies) of the Moldovanu and Winter's (1995) sequential negotiation game with the underlying game in coalitional form being (N, V) .

We now turn to the characterization of stable cartels for the case of costless reentry (subsection 3.1) and of unprofitable reentry (subsection 3.2).

3.1 Stable cartels with costless reentry

Suppose that cost and demand functions are such that at least two firms, say j and k , can be forced to stay out of the market by their rivals i.e. that $\{i, j\}$ and $\{i, k\}$ are feasible. We have seen in subsection 2.2 that, for a given demand function, this situation can arise for a large variety of cost functions. The question is thus to determine the cost characteristics of the firm which is

¹⁴ That this will also hold for an arbitrary number of firms is discussed in the next section.

likely to exit the market or, equivalently, the cost characteristics of the firms which belong to a stable cartel.

To begin with, let the set of feasible cartels be given by $\{\{i, j\}, \{i, k\}, N\}$ and suppose that firm j 's average cost function is lower than firm k 's i.e. $\mu_j < \mu_k$. In this case we have established in Lemma 2 that, for any payoff vector P in the set of attainable payoff vectors¹⁵ for cartel $\{i, k\}$, $\mathcal{V}_\alpha(i, k)$, there exists a payoff vector P' in $\mathcal{V}_\alpha(i, j)$ such that $P'_i > P_i$ and $P'_j > P_j$. Accordingly, $\{i, j\}$ will be able to block any payoff vector in $\mathcal{V}_\alpha(i, k)$ and no payoff vector in $\mathcal{V}_\alpha(i, k)$ belongs to the core of (N, V) . In other words, $\{i, k\}$ cannot be stable. On the other hand, Lemma 1 implies that cartel N cannot be stable since, for any payoff vector in $\mathcal{V}_\alpha(N)$ there exists a payoff vector in $\mathcal{V}_\alpha(i, j)$ which gives more to firms i and j . As a consequence, $\{i, j\}$ is the only cartel which can be stable, that is, cartel $\{i, j\}$ is the unique stable cartel whenever the core of (N, V) , $\mathcal{C}(N, V)$, is non-empty. Now, for a payoff vector P in $\mathcal{V}_\alpha(i, j)$ to belong to $\mathcal{C}(N, V)$, it must give to firm i a payoff at least as large as the maximal payoff firm i can obtain in $\mathcal{V}_\alpha(i, k)$, $v_{\alpha i}^M(i, k)$. Hence, denoting by $\partial\mathcal{V}_\alpha(i, j)$ the set of undominated payoff vectors for firms i and j in $\mathcal{V}_\alpha(i, j)$ i.e. $\partial\mathcal{V}_\alpha(i, j) = \{P \in \mathcal{V}_\alpha(i, j) \mid \nexists P' \in \mathcal{V}_\alpha(i, j) \text{ such that } P'_i > P_i \text{ and } P'_j > P_j\}$, it is quite easy to verify that the core of (N, V) is simply equal to $\{P \in \partial\mathcal{V}_\alpha(i, j) \mid P_i \geq v_{\alpha i}^M(i, k)\}$. The core of (N, V) is therefore non-empty since we have established in Lemma 2 that the maximal payoff firm i can obtain in $\mathcal{V}_\alpha(i, j)$ is strictly greater than $v_{\alpha i}^M(i, k)$. It then follows

Proposition 1 *Let $\mathcal{F} = \{\{i, j\}, \{i, k\}, N\}$ and let reentry be costless, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if $\mu_j < \mu_k$ then $\{i, j\}$ is the unique stable cartel.*

Let us now consider the case where $\{j, k\}$ is also feasible i.e. $\mathcal{F} = \{\{i, j\}, \{i, k\}, \{j, k\}, N\}$ and let us suppose that $\mu_i < \mu_j$, $\mu_i < \mu_k$ and $\mu_j < \mu_k$. Note immediately that, since $\mu_i < \mu_j$, Lemma 2 implies that, for any payoff vector P belonging to $\mathcal{V}_\alpha(j, k)$, there exists a payoff vector P' in $\mathcal{V}_\alpha(i, k)$ such that $P'_i > P_i$ and $P'_k > P_k$. Therefore, the core of (N, V) will have an empty intersection with $\mathcal{V}_\alpha(j, k)$ and, as previously, $\{i, j\}$ is the only cartel which can be stable. Now, since $\{j, k\}$ is feasible and firm k obtains a zero payoff whenever $\{i, j\}$ forms, firms j and k can block any payoff in $\mathcal{V}_\alpha(i, j)$ which does not give to firm j a payoff at least as large as $v_{\alpha j}^M(j, k)$. Hence, we obtain

Proposition 2 *Let $\mathcal{F} = \{\{i, j\}, \{i, k\}, \{j, k\}, N\}$ and let reentry be costless, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if $\mu_i < \mu_j$, $\mu_j < \mu_k$ and $\mu_i < \mu_k$ then $\{i, j\}$ is the unique stable cartel whenever $(v_{\alpha i}^M(i, k), v_{\alpha j}^M(j, k), 0) \in \mathcal{V}_\alpha(i, j)$ while if $(v_{\alpha i}^M(i, k), v_{\alpha j}^M(j, k), 0) \notin \mathcal{V}_\alpha(i, j)$ then no stable cartel exists.*

¹⁵ To make easier the use of our results (see Lemma 1 to 4) of the previous section, we shall work, in all what follows, directly with the set of attainable payoff vectors for a feasible cartel S , $\mathcal{V}_\alpha(S)$, instead of $V(S)$. This should not create any confusion since only feasible cartel matter for the analysis and, for each feasible cartel $V(S) = \mathcal{V}_\alpha(S)$.

These two propositions show that the results for the case of costless reentry confirm those obtained for the perfect competition case as well as the war of attrition's, namely that the firm with the highest average cost function will exit the market. We now show that this prediction will not hold in general whenever reentry is unprofitable.

3.2 Stable cartels with unprofitable reentry

To begin with, let us consider the case where the set of feasible cartels is given by $\{\{i, j\}, \{i, k\}, N\}$. We have shown that, with costless reentry, $\{i, j\}$ is the unique stable cartel whenever $\mu_j < \mu_k$. Now, suppose that, in addition to have a lower average cost function than firm k 's, firm j has also a lower marginal cost function i.e. $\theta_j < \theta_k$. Then, from Lemma 3, we know that, for any payoff vector P in $\mathcal{V}_\alpha(i, j)$, there exists P' in $\mathcal{V}_\alpha(i, k)$ such that $P'_i > P_i$ and $P'_k > P_k$. Hence no payoff vector in $\mathcal{V}_\alpha(i, j)$ belongs to the core of (N, V) and cartel $\{i, j\}$ cannot be stable.

This shows clearly that the prediction about the characteristics of the exiting firm obtained in the costless reentry case will not hold. This conclusion will now be reinforced by showing that $\{i, k\}$ is a stable cartel.

To begin with, from Lemma 4, if P belongs to $\partial\mathcal{V}_\alpha(i, k)$ (the set of attainable payoff vectors for $\{i, k\}$ which are undominated for firms i and k) then there does not exist $P' \in \mathcal{V}_\alpha(N)$ such that $P'_i > P_i$ and $P'_k > P_k$. Accordingly, to show that $\{i, k\}$ is stable, it must be shown that for some payoff vector P in $\mathcal{V}_\alpha(i, k)$ there does not exist a payoff vector P' in $\mathcal{V}_\alpha(i, j)$ such that $P'_i > P_i$ and $P'_k > P_k$. But at any payoff vector in $\mathcal{V}_\alpha(i, k)$, firm j 's payoff is zero so that $\{i, k\}$ is stable whenever there exists a payoff vector in $\mathcal{V}_\alpha(i, k)$ which gives to firm k a payoff larger than zero and to firm i a payoff larger than its maximal payoff in $\mathcal{V}_\alpha(i, j)$ i.e. $v_{\alpha i}^M(i, j)$. Now, from Lemma 3, we know that $v_{\alpha i}^M(i, k)$ is strictly greater than $v_{\alpha i}^M(i, j)$ for α sufficiently close to 1, while firm k 's minimal payoff in $\mathcal{V}_\alpha(i, k)$ exceeds $v_k(i, k)$ which, by assumption 5, is strictly positive. Therefore, $\{P \in \mathcal{V}_\alpha(i, k) | P_i \geq v_{\alpha i}^M(i, j)\}$ will be non-empty and is included in the core of (N, V) so that cartel $\{i, k\}$ is stable.

When the set of feasible cartels is given by $\{\{i, j\}, \{i, k\}, N\}$, the unprofitable reentry case differs from the costless reentry one in another respect. We shall indeed show that, with unprofitable reentry, cartel N can be stable while, with costless reentry, we have established that $\{i, j\}$ is the unique stable cartel. To see this, recall the discussion in subsection 2.4 and remark that firm i 's maximal payoff in $\mathcal{V}_\alpha(N)$ i.e. $v_{\alpha i}^M(N)$ can exceed $v_{\alpha i}^M(i, k)$. But if $v_{\alpha i}^M(N) \geq v_{\alpha i}^M(i, k)$, any payoff vector in $\partial\mathcal{V}_\alpha(N)$ which gives to firm i a payoff at least as large as $v_{\alpha i}^M(i, k)$ will belong to the core of (N, V) so that N is stable. Hence we obtain

Proposition 3 *Let $\mathcal{F} = \{\{i, j\}, \{i, k\}, N\}$, let $\theta_j < \theta_k$ and let reentry be unprofitable, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, either $v_{\alpha i}^M(N) < v_{\alpha i}^M(i, k)$ and $\{i, k\}$ is the unique stable cartel, or $v_{\alpha i}^M(N) \geq v_{\alpha i}^M(i, k)$ and both $\{i, k\}$ and N are stable.*

Before we comment on this Proposition, let us consider the case where the set of feasible cartels is given by $\{\{i, j\}, \{i, k\}, \{j, k\}, N\}$. We have¹⁶

Proposition 4 *Let $\mathcal{F} = \{\{i, j\}, \{i, k\}, \{j, k\}, N\}$, let $\theta_i < \theta_j < \theta_k$ and let reentry be unprofitable, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, $\{j, k\}$ is the unique stable cartel whenever $(0, v_{\alpha j}^M(i, j), v_{\alpha k}^M(i, k)) \in \mathcal{V}_\alpha(j, k)$ while if $(0, v_{\alpha j}^M(i, j), v_{\alpha k}^M(i, k)) \notin \mathcal{V}_\alpha(j, k)$ then no stable cartel exists.*

Three basic conclusions are obtained from the results stated in Proposition 3 and 4. First, as discussed in subsection 2.2, a two-firm cartel is feasible whenever the third firm has a sufficiently high average cost function. This can arise with this firm having large fixed costs and a low marginal cost function or a high marginal cost function and large fixed costs. Accordingly, a large variety of firms' cost functions can lead to the same set of feasible cartels. Now, our results show that the determination of which cartel is *stable* involves only a comparison between firms' marginal cost function. Consequently, when reentry is unprofitable, firms' fixed costs only matter for the determination of the set of feasible cartels. Second, if the firm with the highest marginal cost belongs to a feasible two-firm cartel, then this firm belongs to a stable cartel (if such a cartel exists) and will thus remain on the market. In particular, when all two-firm cartels are feasible, if a stable cartel exists then the two firms with higher marginal cost functions survive i.e. the most cost efficient firm can exit the market¹⁷. Finally, the possibility of predation will not imply by itself that predation will occur since we have seen that N can be stable.

4 An extension: Stable cartels with an arbitrary number of firms

The purpose of this section is that of generalizing the analysis to the case of an arbitrary number of firms. To begin with, let us define a feasible cartel: $S \subseteq N$ is said feasible if and only if there exists an equilibrium σ of Γ_α and $\tau < +\infty$ such that, along the path generated by σ , at each period $t \geq \tau$ all firms in S stay in and all firms in $N \setminus S$ stay out of the market. To each feasible cartel, as in the production game of Section 2, is associated a set

¹⁶ Since $\theta_i < \theta_j < \theta_k$, Lemma 3 implies, on the one hand, that for any P in $\mathcal{V}_\alpha(i, j)$ there exists P' in $\mathcal{V}_\alpha(j, k)$ such that $P'_j > P_j$ and $P'_k > P_k$ and, on the other hand, that for any P in $\mathcal{V}_\alpha(i, k)$ there exists P' in $\mathcal{V}_\alpha(j, k)$ such that $P'_j > P_j$ and $P'_k > P_k$. In other words, cartel $\{j, k\}$ can block any payoff vectors which can be proposed by cartels $\{i, j\}$ and $\{i, k\}$. Hence, $\{j, k\}$ is the only two-firm cartel which can be stable. Now, for a vector P in $\mathcal{V}_\alpha(j, k)$ to be in the core of (N, V) , it must give to firms j and k a payoff at least as large as $v_{\alpha j}^M(i, j)$ and $v_{\alpha k}^M(i, k)$ respectively. Therefore, $\{j, k\}$ is stable whenever $(0, v_{\alpha j}^M(i, j), v_{\alpha k}^M(i, k))$ belongs to $\mathcal{V}_\alpha(j, k)$. Finally, it is easy to verify that, when all two-firm cartels are feasible, cartel N cannot be stable.

¹⁷ The condition under which a stable cartel exists when all two-firm cartels are feasible, namely that $(0, v_{\alpha j}^M(i, j), v_{\alpha k}^M(i, k)) \in \mathcal{V}_\alpha(j, k)$, is more likely to be satisfied when θ_i is small since, as shown by Lemma 3, a low θ_i implies low values for $v_{\alpha j}^M(i, j)$ and $v_{\alpha k}^M(i, k)$. This also contrasts with the case of costless reentry where the presence of a firm with high average costs makes it more likely the existence of a stable cartel.

$\mathcal{V}_\alpha(S)$ of equilibrium payoff vectors. Then proceeding as in subsections 2.3 and 2.4, it is easy to verify that the analogue of Claim 1 and Claim 2 writes as

Claim 3 *Let S be a feasible cartel and let reentry be costless. Then,*

1. $\forall \alpha \in (0, 1), \mathcal{V}_\alpha(S) \subseteq \{(P_i)_{i \in N} | (P_i)_{i \in S} \in \mathcal{A}(S), P_i \geq \max\{0, v_i(N)\} \forall i \in S, P_j = 0 \forall j \in N \setminus S\},$
2. $\forall P^0 \in \{(P_i)_{i \in N} | (P_i)_{i \in S} \in \mathcal{A}(S), P_i > \max\{0, v_i(N)\} \forall i \in S, P_j = 0 \forall j \in N \setminus S\},$ *there exists $\alpha^0 < 1$ such that, for all $\alpha \in (\alpha^0, 1), P^0 \in \mathcal{V}_\alpha(S).$*

Claim 4 *Let S be a feasible cartel and let reentry be unprofitable. Then,*

1. $\forall \alpha \in (0, 1), \mathcal{V}_\alpha(S) \subseteq \{P | (P_i)_{i \in S} \in \mathcal{A}(S), P_i \geq \max\{0, v_i(S)\} \forall i \in S \text{ and } P_k = 0 \forall k \in N \setminus S\},$
2. $\forall P^0 \in \{P | (P_i)_{i \in S} \in \mathcal{A}(S), P_i > \max\{0, v_i(S)\} \forall i \in S \text{ and } P_k = 0 \forall k \in N \setminus S\},$ *there exists $\alpha^0 < 1$ such that, for all $\alpha \in (\alpha^0, 1), P^0 \in \mathcal{V}_\alpha(S).$*

Remark that the only difference with the three firms case is that $v_i(S)$ can now be negative whenever $S \neq N$, while with three firms our hypothesis of non-monopolization (assumption 5) implies that only $v_i(N)$ could be negative. Finally, the game in coalitional form (N, V) is defined as in section 3, that is, $V(S) = \{P \in R_+^N | P_i = 0 \forall i \in S\}$ whenever S is not feasible and, for a feasible cartel $S \subseteq N$,

$$V(S) = \cup_{S_h \in \{Z | Z \subseteq S, Z \in \mathcal{F}\}} \mathcal{V}_\alpha(S_h).$$

At the outset, we consider the question of the non-emptiness of the core of (N, V) .

Proposition 5 *A sufficient condition for the core of (N, V) to be non-empty is that there exists $S^0 \neq \emptyset$ such that $S^0 \subseteq S$ for all $S \in \mathcal{F}$.*

This result (proven in the Appendix) shows that if at least one firm always finds profitable to remain on the market, then a stable cartel exists¹⁸. Note also that, whenever α is sufficiently close to 1, this condition reduces simply to the existence of at least one firm, say i , for which its minimax in the Cournot game with a set N of active firms, $v_i(N)$, is strictly positive.

We shall denote by S^* a stable cartel. Remark immediately that if a stable cartel exists then it has a non-empty intersection with all feasible cartels, that is, $S^* \cap S \neq \emptyset$ for all $S \in \mathcal{F}$. Indeed, for all $P \in \mathcal{V}_\alpha(S^*)$, we have that $P_i = 0$ for all $i \in N \setminus S^*$. Hence, if $S^* \cap S = \emptyset$ for some $S \in \mathcal{F}$, then $S \subseteq N \setminus S^*$ and for all $P \in \mathcal{V}_\alpha(S^*)$ there exists $P' \in \mathcal{V}_\alpha(S)$ such that $P'_i > P_i = 0$ for all $i \in S$.

¹⁸ In the Appendix, we show that this condition is also sufficient to ensure the non-emptiness of the core of every subgame (S, V^S) of (N, V) , where $S \subseteq N$ and V^S stands for the restriction of V to the subsets of S . Accordingly, under this condition, we can continue to use Moldovanu and Winter's (1995) results to motivate our definition of a stable cartel.

The core of (N, V) will thus have an empty intersection with $\mathcal{V}_\alpha(S^*)$, that is, S^* will not be stable.

Define then the following vectors of firms' characteristics. Let the vector of average cost functions $\mu(S) = (\mu_i)_{i \in S}$ be ordered in such a way that $\mu_i \leq \mu_j$ if and only if $i < j$ for all $i, j \in S$. In the same way, let $\theta(S) = (\theta_i)_{i \in S}$ and $\bar{q}(S) = (\bar{q}_i)_{i \in S}$ be ordered so that $\theta_i \leq \theta_j$ and $\bar{q}_i \leq \bar{q}_j$ if and only if $i < j$ for all $i, j \in S$. Now the following propositions (proven in the Appendix) give a characterization of stable cartels for the case of costless reentry and for that of unprofitable reentry.

Proposition 6 *Let reentry be costless, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then we have:*

1. *if $\bar{q}_i = \bar{q}$ for all $i \in N$, then there does not exist $S \in \mathcal{F}$ such that either $|S| < |S^*|$, or $|S| = |S^*|$ and $^{19} \mu(S) < \mu(S^*)$;*
2. *if $\bar{q}_i \neq \bar{q}_j$ for some $i, j \in N$, then there does not exist $S \in \mathcal{F}$ such that $|S| = |S^*|$ and $\mu(S) < \mu(S^*)$.*

Let $\mathcal{V}(S) = \{(P_i)_{i \in N} | (P_i)_{i \in S} \in \mathcal{A}(S), P_i > \max\{0, v_i(S)\} \forall i \in S, P_i = 0 \forall i \in N \setminus S\}$, then

Proposition 7 *Let reentry be unprofitable and suppose that, for any S and S' , $S' \subset S$ implies that $\forall P \in \mathcal{V}(S)$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P_i$ for all $i \in S'$, then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then we have:*

1. *if $\bar{q}_i = \bar{q}$ for all $i \in N$, then there does not exist $S \in \mathcal{F}$ such that $|S| = |S^*|$ and $\theta(S) > \theta(S^*)$;*
2. *if $\theta_i = \theta$ for all $i \in N$, then there does not exist $S \in \mathcal{F}$ such that $|S \setminus S^*| \geq 2$, $|S| = |S^*|$ and $\bar{q}(S^* \setminus S) < \bar{q}(S \setminus S^*)$.*

Accordingly, whenever capacities are identical across firms, Proposition 6 and 7 provide the same qualitative results than those given in Proposition 1 and 3. This simply follows from the fact that, with identical capacities, part (b) of Lemma 2 and 3 naturally extends to the case of an arbitrary number of firms.

In the case of costless reentry, the presence of differences in capacities introduces the following possibility. Consider an example with $N = \{1, 2, 3, 4\}$. Suppose that firm 4 has the largest fixed cost but also the largest capacity so that $\{1, 4\}$ is a feasible cartel. If firms 2 and 3 have sufficiently small capacities it is possible that neither $\{1, 2\}$ nor $\{1, 3\}$ is feasible while $\{1, 2, 3\}$ is feasible. Then $\{1, 4\}$ can be stable since it could be possible for 1 to obtain more in $\mathcal{V}_\alpha(1, 4)$ than in $\mathcal{V}_\alpha(1, 2, 3)$. Accordingly, although reentry is costless, differences in capacities lead to the possibility that the firm with the highest average cost function remains on the market.

¹⁹ For two s -dimensional vectors x and y , $x < y$ means that $x_i \leq y_i$ for $i = 1, \dots, s$ with a strict inequality for at least one i .

With unprofitable reentry, we have seen that the characterization of a stable cartel relies on the firms' minimax payoff in the Cournot game. However, in the case of an arbitrary number of firms, firm i 's minimax payoff, $v_i(S)$, is given by

$$v_i(S) = \pi_i \left(q_i^R \left(\sum_{j \in S \setminus i} \bar{q}_j \right), \sum_{j \in S \setminus i} \bar{q}_j \right)$$

and will in general depend upon θ_i as well as on the vector of capacities. This explains why in order to extend Lemma 3 to the case of an arbitrary number of firms the condition that capacities are identical is introduced. Now to understand the role of capacities when the θ s are identical, let us take an example. Consider a situation where cartels $\{1, 2, 3\}$ and $\{1, 4, 5\}$ are feasible with $\bar{q}_2 \geq \bar{q}_3$ and $\bar{q}_4 \geq \bar{q}_5$. Then Proposition 7 tells us that if cartel $\{1, 2, 3\}$ is stable then we cannot have $\bar{q}_4 \geq \bar{q}_2$ and $\bar{q}_5 \geq \bar{q}_3$ with a least one strict inequality. To see why it must be so, suppose to the contrary that $\bar{q}_4 > \bar{q}_2$ and $\bar{q}_5 \geq \bar{q}_3$. By the definition of firm i 's minimax $v_i(S)$, $\bar{q}_5 \geq \bar{q}_3$ implies that $v_4(1, 4, 5) + F_4 \leq v_2(1, 2, 3) + F_2$ while $\bar{q}_4 > \bar{q}_2$ implies that $v_5(1, 4, 5) + F_5 < v_3(1, 2, 3) + F_3$. Therefore firm 1's maximal payoff in $\mathcal{V}_\alpha(1, 4, 5)$ is strictly greater than in $\mathcal{V}_\alpha(1, 2, 3)$ since all firms have the same marginal cost function and firm 1's maximal payoff in $\mathcal{V}_\alpha(1, 2, 3)$, resp. in $\mathcal{V}_\alpha(1, 4, 5)$, does not depend upon the fixed cost of firms 2 and 3, resp. 4 and 5. Consequently, cartel $\{1, 2, 3\}$ cannot be stable since any payoff vector in $\mathcal{V}_\alpha(1, 2, 3)$ will be blocked by cartel $\{1, 4, 5\}$.

Remark incidentally that if, like in Ghemawat and Nalebuff (1985), firms' fixed cost is an increasing function of their capacity while marginal costs are identical, then the result we obtain with unprofitable reentry goes in the opposite direction of the one obtained in a war of attrition.

5 Concluding remarks

Using the concept of a stable cartel, we have obtained two predictions on the cost characteristics of the exiting firms in a infinite horizon production game. Loosely speaking, whenever reentry is costless, the exiting firms are those with higher average cost functions while, whenever reentry is unprofitable, the exiting firms are those with lower marginal cost functions. Accordingly, a firm with a low average and marginal cost function will be expected to stay in the market when reentry is costless but will be expected to exit the market when reentry is unprofitable. Note that reentry costs are sunk costs. Consequently our analysis points out a novel implication of sunk costs, namely, sunk costs matter for the composition of the industry.

This sharp difference between the results obtained with costless reentry and with unprofitable reentry has also been shown to come from the incidence of the assumption about reentry on the minimal payoff that a firm can achieve by staying in the market.

To see this point clearly, let us consider the three firms case where cartels $\{i, j\}$ and $\{i, k\}$ are the only two-firms cartels to be feasible. Let us furthermore suppose that firm j has lower marginal and average cost functions than firm k 's. When reentry is costless, the minimal payoff of firms j and k will both tend to zero whenever the discount factor tends to one. The comparison between the maximal payoff that firm i can achieve by remaining on the market with firm j and the one it can obtain by staying in the market with firm k will thus rely only upon what we have called the "efficiency effect". This efficiency effect tells us simply that, if firms j and k must receive the same payoff when they stay in the market with firm i , then firm i can obtain a larger payoff by staying in with the firm having the lowest average cost function, that is, firm j . Now, firm j obtains a zero payoff when it stays out of the market while it achieves a strictly positive one by staying in with firm i . Therefore, firms i and j can both obtain a larger payoff by staying together in the market than the one they can obtain if it is firms i and k which stay in. In other words, cartel $\{i, k\}$ cannot be stable.

When reentry is unprofitable we have shown that, as the discount factor tends to one, the minimal payoff that a firm, say j , can obtain by staying in the market with firm i tends to its minimax payoff in the Cournot game played by firms i and j . This minimax payoff²⁰ depends obviously on the firm's cost characteristics with two main consequences. First, the maximal payoff that firm i can obtain by staying in the market with firm j will no longer depend on firm j 's fixed costs. Accordingly, the comparison between firm i 's maximal payoff when it stays in with firm j and when it stays in with firm k relies only upon differences among the marginal cost function of firms j and k . Second, the lower a firm's marginal cost function is, the larger its minimax payoff is. Hence, since firm j is supposed to have a lower marginal cost function than firm k , it has also a larger minimax payoff. This induces a "minimax effect": everything being equal, firm i can obtain a larger payoff by staying in the market with the firm having the smaller minimax payoff. This effect will therefore work in an opposite direction to the efficiency effect (everything being equal, the lower the marginal cost function is the lower the average cost function). Moreover we can show that the minimax effect dominates the efficiency effect so that, the maximal payoff that firm i can obtain by staying in with firm j is strictly smaller than the one it can obtain by staying in with firm k . Now, since firm k will obtain its minimax payoff if it stay in with firm i while it obtains a zero payoff if firms i and j stay in the market, firms i and k will thus obtain a larger payoff when they stay together in the market than the one they can obtain if firms i and j remain on the market. Consequently, cartel $\{i, j\}$ cannot be stable.

²⁰ These minimax payoffs are strictly positive under our assumption of non-monopolization of the market.

Appendix

Proof of Lemma 3

To save on notations, let $\hat{q}_h(q_i)$, for $h \neq i$, be equal to $\min\{q_h | \pi_h(q_h, q_i) = v_h(i, h)\}$. Accordingly, $\Pi_i^*(v_h(i, h), h)$ is simply equal to $\max_{q_i \in X_i} \pi_i(q_i, \hat{q}_h(q_i))$.

Note first that the definition of $\hat{q}_h(q_i)$ and assumptions 1 and 3 imply that, for $h = j, k$, we have (i) $\hat{q}_h(\bar{q}_i) = q_h^R(\bar{q}_i)$; (ii) $\partial \pi_h(\hat{q}_h(q_i), q_i) / \partial q_h > 0 \quad \forall q_i \in [0, \bar{q}_i]$; and (iii) $\hat{q}_h(q_i)$ is strictly increasing in q_i for $q_i \in [0, \bar{q}_i]$. Now, from the definition of $q_h^R(q_i)$, we also have that

$$\frac{dq_h^R(\bar{q}_i)}{d\theta_h} = \frac{\partial^2 c(q_h^R(\bar{q}_i); \theta_h, \bar{q}_h) / \partial q_h \partial \theta_h}{\partial^2 \pi_h(q_h^R(\bar{q}_i), \bar{q}_i) / \partial q_h^2}$$

which is strictly negative. Hence, $\hat{q}_j(\bar{q}_i) = q_j^R(\bar{q}_i) > q_k^R(\bar{q}_i) = \hat{q}_k(\bar{q}_i)$ if and only if $\theta_j < \theta_k$. On the other hand, for any $q_i \in [0, \bar{q}_i]$, it is easy to verify that

$$\frac{d\hat{q}_h(q_i)}{d\theta_h} = \frac{\partial c(\hat{q}_h(q_i); \theta_h, \bar{q}_h) / \partial \theta_h - \partial c(q_h^R(\bar{q}_i); \theta_h, \bar{q}_h) / \partial \theta_h}{\partial \pi_h(\hat{q}_h(q_i), q_i) / \partial q_h}$$

Note that, for any $q_i \in [0, \bar{q}_i]$, the denominator of this expression is strictly positive from property (ii) above while the numerator is strictly negative since, by property (iii) above, $\hat{q}_h(q_i) < \hat{q}_h(\bar{q}_i) = q_h^R(\bar{q}_i)$ and $\partial^2 c(q_h; \theta, \bar{q}_h) / \partial q_h \partial \theta_h$ has been supposed strictly positive in assumption 1. Therefore, for all $q_i \in [0, \bar{q}_i]$, $d\hat{q}_h(q_i) / d\theta_h$ is strictly negative.

We thus obtain that, for all $q_i \in [0, \bar{q}_i]$, $\hat{q}_j(q_i) > \hat{q}_k(q_i)$ if and only if $\theta_j < \theta_k$. Furthermore, since π_i is a decreasing function of Q_{-i} , we have that, for all $q_i \in [0, \bar{q}_i]$, $\pi_i(q_i, \hat{q}_j(q_i)) < \pi_i(q_i, \hat{q}_k(q_i))$ if and only if $\theta_j < \theta_k$. The result stated in the part (a) of the Lemma then follows from the definition of Π_i^* and the application of the Folk Theorem.

Proof of Proposition 5

Let $v^M(S^0; S)$ be such that, for all $P \in \mathcal{V}_\alpha(S)$, $\sum_{i \in S^0} P_i \leq v^M(S^0; S)$ with S^0 being such that if $S \in \mathcal{F}$ then $S^0 \subseteq S$. Now, since \mathcal{F} is clearly finite, there exists $S^* \in \mathcal{F}$ such that $v^M(S^0; S^*) \geq v^M(S^0; S) \quad \forall S \in \mathcal{F}$. We claim that S^* is stable.

Suppose to the contrary that S^* is not stable, that is, that $\forall P \in \mathcal{V}_\alpha(S^*)$ there exists $S' \subseteq N$ and $P' \in \mathcal{V}_\alpha(S')$ such that $P'_i > P_i \quad \forall i \in S'$. Clearly such S' must be feasible and therefore, by assumption, $S^0 \subseteq S'$. Hence, if S^* is not stable then for some $S' \in \mathcal{F}$ and $P' \in \mathcal{V}_\alpha(S')$ we have that $\sum_{i \in S^0} P'_i > v^M(S^0; S^*)$. But this contradicts the definition of S^* .

This kind of argument can easily be used to show that, as claimed in footnote 18, the condition of the Proposition is sufficient for every subgame (S, V^S) of (N, V) , where V^S stands for the restriction of V to all subsets of S , to have a non-empty core.

Proof of Proposition 6

We first prove that there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then there does not exist $S \in \mathcal{F}$ with $S \cap S^* \neq \emptyset$ and $|S| = |S^*|$ such that $\mu(S) < \mu(S^*)$. This comes from the following.

Lemma 5 *Suppose that reentry is costless. If S, S' are feasible with $|S| = |S'|$, $S \cap S' \neq \emptyset$ and $\mu(S') < \mu(S)$ then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$ and for all $P \in \mathcal{V}_\alpha(S)$, there exists $P' \in \mathcal{V}_\alpha(S')$ such that $P'_i > P_i \forall i \in S'$.*

Proof. (a) Let $D = \{0, 1, \dots, \bar{d}\}$ with $\bar{d} = |S \setminus S'|$ and let $d \in D$. Then define S_d as follows:

- (i) $S_0 = S$
- (ii) $S_d = (S_{d-1} \setminus \{h(d)\}) \cup \{l(d)\}$ with $l(d) \in S' \setminus S$, $h(d) \in S \setminus S'$ and $d \geq 1$

Obviously $S_{\bar{d}} = S'$ and $\{S_d\}_{d \in D}$ is a sequence of subsets of N which starts from S and converges to S' . Remark also that $S_d \setminus S_{d-1} = \{l(d)\}$, $S_{d-1} \setminus S_d = \{h(d)\}$ and $|S_d| = |S_{d-1}|$ for any $d \geq 1$. Furthermore, since $\mu(S') < \mu(S)$ by assumption, then $\mu(S' \setminus S) < \mu(S \setminus S')$ and therefore there exists a sequence $\{\tilde{S}_d\}_{d \in D}$ such that, for all $d \geq 1$, $\mu_{l(d)} \leq \mu_{h(d)}$ with a strict inequality for a least one d . As a consequence, we have that $\mu(\tilde{S}_d) < \mu(\tilde{S}_{d-1})$ for any $d \geq 1$.

Now, for any $d \in D$, let us define $\bar{\mathcal{V}}^c(\tilde{S}_d)$ as follows:

$$\bar{\mathcal{V}}^c(\tilde{S}_d) = \{(P_i)_{i \in N} | (P_i)_{i \in \tilde{S}_d} \in \mathcal{A}(\tilde{S}_d), P_i = 0 \forall i \in N \setminus \tilde{S}_d, \\ P_i \geq \max\{0, v_i(N)\} \forall i \in \tilde{S}_d\}$$

Note immediately that since S and S' are feasible then, $\forall i \in N \setminus (S \cap S')$, $v_i(N) \leq 0$ and hence $\max\{0, v_i(N)\} = 0$. This implies that there exist payoff vectors in $\bar{\mathcal{V}}^c(\tilde{S}_d)$, resp. in $\bar{\mathcal{V}}^c(\tilde{S}_{d-1})$, such that $P_{l(d)} = 0$, resp. $P_{h(d)} = 0$.

(b) Then we can proceed as in the discussion preceding Lemma 2 to establish that, for all $d \geq 1$, we have that for all $P \in \bar{\mathcal{V}}^c(\tilde{S}_{d-1})$ there exists $P' \in \bar{\mathcal{V}}^c(\tilde{S}_d)$ such that $P'_i \geq P_i \forall i \in \tilde{S}_d$ whenever $\mu_{l(d)} \leq \mu_{h(d)}$ and $P'_i > P_i \forall i \in \tilde{S}_d$ whenever $\mu_{l(d)} < \mu_{h(d)}$. Since the latter holds for at least one d , we therefore obtain that $\forall P \in \bar{\mathcal{V}}^c(S)$ there exists $P' \in \bar{\mathcal{V}}^c(S')$ such that $P'_i > P_i$ for all $i \in S'$. Consequently, by continuity, we have that $\forall P \in \{P \in \bar{\mathcal{V}}^c(S) | P_i > \max\{0, v_i(N)\} \forall i \in S\}$ there exists $P' \in \{P \in \bar{\mathcal{V}}^c(S') | P_i > \max\{0, v_i(N)\} \forall i \in S'\}$ such that $P'_i > P_i \forall i \in S'$ and the result then follows from the application of the Folk Theorem.

It just remains to prove that, whenever $\bar{q}_i = \bar{q}$ for all $i \in N$, there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then there does not exist $S \in \mathcal{F}$ such that $|S| < |S^*|$. Let $\hat{S}(s)$ be such that $|\hat{S}(s)| = s$ and, for all $i \in \hat{S}(s)$, $\mu_i < \mu_j \forall j \in N \setminus \hat{S}(s)$. Now the assumption that $\bar{q}_i = \bar{q}$ for all $i \in N$ implies that $v_i(S \cup \{i\}) = \pi_i(q_i^R(|S|\bar{q}), |S|\bar{q})$ and consequently if S with $|S| = s$ is feasible then $\hat{S}(s)$ is also feasible. Furthermore, if there exists $S \in \mathcal{F}$ with $|S| = s < s^* = |S^*|$ then $\hat{S}(s) \in \mathcal{F}$ and $\hat{S}(s) \subset \hat{S}(s^*)$. Therefore, by Lemma 5, there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then $S^* = \hat{S}(s^*)$ and, consequently, if in addition there exists $S \in \mathcal{F}$ such that $|S| < |S^*|$ then

$\hat{S}(s) \subset S^*$. The result then follows since, with costless reentry, it is clear that if S, S' are feasible with $S' \subset S$ then there exists $\bar{\alpha} < 1$ such that for all $\alpha \in (\bar{\alpha}, 1)$ and $\forall P \in \mathcal{V}_\alpha(S)$, there exists $P' \in \mathcal{V}_\alpha(S')$ such that $P'_i > P_i \forall i \in S'$.

Proof of Proposition 7

1. Remark first that the assumption that, for any S and $S', S' \subset S$ implies that $\forall P \in \mathcal{V}(S)$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P_i$ for all $i \in S'$, entails immediately that if S^* is stable then no feasible cartel is included in S^* or, equivalently, that $v_i(S^*) \geq 0 \forall i \in S^*$.

We then extend Lemma 3 as follows:

Lemma 6 *Suppose that reentry is unprofitable and $\bar{q}_i = \bar{q} \forall i \in N$. Then, for any S and S' such that $v_i(S) \geq 0 \forall i \in S$; $v_i(S') \geq 0 \forall i \in S'$; $|S| = |S'|$; $S \cap S' \neq \emptyset$; and $\theta(S) < \theta(S')$ we have: (a) for all $P \in \mathcal{V}(S)$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P_i$ for all $i \in S'$ and (b) if in addition S and S' are feasible then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$ and for all $P \in \mathcal{V}_\alpha(S)$, there exists $P' \in \mathcal{V}_\alpha(S')$ such that $P'_i > P_i$ for all $i \in S'$.*

The proof of these results follows quite closely the one of Lemma 5 and is therefore omitted. Note that this Lemma implies that, for α sufficiently close to one, if S^* is stable then, whenever $\bar{q}_i = \bar{q}$, there does not exist $S \in \mathcal{F}$ such that $v_i(S) \geq 0 \forall i \in S$, $|S| = |S^*|$ and $\theta(S) > \theta(S^*)$.

Hence, to obtain the result stated in the first part of Proposition 7, it remains to prove that there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then there does not exist $S \in \mathcal{F}$ such that $v_i(S) < 0$ for some $i \in S$, $|S| = |S^*|$, $S \cap S^* \neq \emptyset$ and $\theta(S) > \theta(S^*)$. Note that if S is feasible and $v_i(S) < 0$ for some $i \in S$ then there exists $S' \in \mathcal{F}$ such that $S' \subset S$ and $v_i(S') \geq 0$ for all $i \in S'$. Furthermore, since S^* is stable and S' is a feasible cartel, we have that $S^* \cap S' \neq \emptyset$. Then, since $\theta(S) > \theta(S^*)$, there exists $S'' \subset S^*$ (with $S'' \notin \mathcal{F}$) such that $v_i(S'') \geq 0$, $|S''| = |S'|$, $S'' \cap S' \neq \emptyset$ and $\theta(S') > \theta(S'')$. Now, by the first part of Lemma 6, we obtain that, $\forall P'' \in \mathcal{V}(S'')$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P''_i$ for all $i \in S'$ while, by assumption, we have that $\forall P^* \in \mathcal{V}(S^*)$ there exists $P'' \in \mathcal{V}(S'')$ such that $P''_i > P^*_i$ for all $i \in S''$. Consequently, $\forall P^* \in \mathcal{V}(S^*)$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P^*_i$ for all $i \in S'$ and the result follows from the Folk Theorem.

2. We first prove that, under the assumptions of Proposition 7-2, there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then there does not exist $S \in \mathcal{F}$ such that $v_i(S) \geq 0$ for all $i \in S$; $S \cap S^* \neq \emptyset$; $|S| = |S^*|$; $|S \setminus S^*| \geq 2$; and $\bar{q}(S \setminus S^*) > \bar{q}(S^* \setminus S)$. This is an immediate consequence of the following.

Lemma 7 *Suppose the assumptions of Proposition 7-2 hold. For any S and S' such that $v_i(S) \geq 0 \forall i \in S$; $v_i(S') \geq 0 \forall i \in S'$; $S \cap S' \neq \emptyset$; $|S| = |S'|$; $|S \setminus S'| \geq 2$; and $\bar{q}(S \setminus S') > \bar{q}(S' \setminus S)$ we have: (a) for all $P \in \mathcal{V}(S)$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P_i \forall i \in S'$; and (b) if in addition S and S' are feasible then there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$ and for all $P \in \mathcal{V}_\alpha(S)$, there exists $P' \in \mathcal{V}_\alpha(S')$ such that $P'_i > P_i \forall i \in S'$.*

Proof. First of all, for all S and S' satisfying the requirements of the Lemma and for all $Q \in [0, \sum_{k \in S \cap S'} \bar{q}_k]$, let us define the following.

$$\hat{Q}(S \setminus S', \bar{q}(S \setminus S'), Q) = \min_{(q_i)_{i \in S \setminus S'}} \left\{ \sum_{i \in S \setminus S'} q_i \text{ s.t. } \pi_i \left(q_i, Q + \sum_{j \in S \setminus S' \setminus i} q_j \right) \geq v_i(S) \quad \forall i \in S \setminus S' \right\}$$

where $v_i(S) = \pi_i(q_i^R(\sum_{j \in S \setminus i} \bar{q}_j), \sum_{j \in S \setminus i} \bar{q}_j)$ and $q_i^R(Q_{-i}) = \arg \max_{q_i} \pi_i(q_i, Q_{-i})$. It must be immediately noted that if $\bar{q}(S \setminus S') = \bar{q}(S' \setminus S)$ then $\hat{Q}(S \setminus S', \bar{q}(S \setminus S'), Q) = \hat{Q}(S' \setminus S, \bar{q}(S' \setminus S), Q)$ ²¹. Furthermore, using the envelope theorem and denoting by λ_i the Kuhn and Tucker multiplier associated to the constraint $\pi_i \geq v_i(S)$, we have, for any $i \in S \setminus S'$,

$$\frac{d\hat{Q}(S \setminus S', \bar{q}(S \setminus S'), Q)}{d\bar{q}_i} = \sum_{k \in S \setminus S' \setminus i} \lambda_k \frac{\partial v_k(S)}{\partial \bar{q}_i}$$

Accordingly, since $|S \setminus S'|$ is assumed to be greater or equal to 2, $d\hat{Q}/d\bar{q}_i$ is strictly negative. Hence if S and S' are such that $\bar{q}(S \setminus S') < \bar{q}(S' \setminus S)$ then $\hat{Q}(S \setminus S', \bar{q}(S \setminus S'), Q) > \hat{Q}(S' \setminus S, \bar{q}(S' \setminus S), Q)$ for all $Q \in [0, \sum_{k \in S \cap S'} \bar{q}_k]$. Therefore, since π_k is strictly decreasing in Q_{-k} , we have that, for all $k \in S \cap S'$ and all $(q_k)_{k \in S \cap S'} \in \times_{k \in S \cap S'} X_k$,

$$\begin{aligned} \pi_k \left(q_k, \sum_{l \in (S \cap S') \setminus k} q_l + \hat{Q} \left(S \setminus S', \bar{q}(S \setminus S'), \sum_{l \in S \cap S'} q_l \right) \right) \\ < \pi_k \left(q_k, \sum_{l \in (S \cap S') \setminus k} q_l + \hat{Q} \left(S' \setminus S, \bar{q}(S' \setminus S), \sum_{l \in S \cap S'} q_l \right) \right) \end{aligned}$$

The first part of Lemma 7 then follows by continuity of π_i and this in turn leads, by using the Folk Theorem, to the second result stated in the Lemma.

Now, to complete the proof of Proposition 7, it remains to prove that there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$, if S^* is stable then there does not exist $S \in \mathcal{F}$ such that $v_i(S) < 0$ for some $i \in S$; $S \cap S^* \neq \emptyset$; $|S| = |S^*|$; $|S \setminus S^*| \geq 2$; and $\bar{q}(S \setminus S^*) > \bar{q}(S^* \setminus S)$. Suppose to the contrary that there exists S satisfying these requirements. Since S is feasible and $v_i(S) < 0$ for some $i \in S$, then there exists $S' \in \mathcal{F}$ such that $S' \subset S$ and $v_i(S') \geq 0 \quad \forall i \in S'$. Note also that if S^* is stable then S^* has a nonempty intersection with any feasible cartel and, in particular, $S^* \cap S' \neq \emptyset$. Furthermore, since $\bar{q}(S \setminus S^*) > \bar{q}(S^* \setminus S')$, there exists $S'' \subset S^*$ (with $S'' \notin \mathcal{F}$) such that $|S''| = |S'|$; $S'' \cap S' \neq \emptyset$ and $\bar{q}(S' \setminus S'') > \bar{q}(S'' \setminus S')$. Therefore, using Lemma 7, we have that $\forall P'' \in \mathcal{V}(S'')$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P''_i \quad \forall i \in S'$. Furthermore, by assumption, we also have that $\forall P^* \in \mathcal{V}(S^*)$ there exists

²¹ This follows from the facts that the constraints in the minimization problem are binding at a solution so that fixed costs do not affect \hat{Q} and that we have assumed that $\theta_i = \theta \quad \forall i \in N$.

$P'' \in \mathcal{V}(S'')$ such that $P'_i > P_i^* \forall i \in S''$ and consequently, $\forall P^* \in \mathcal{V}(S^*)$ there exists $P' \in \mathcal{V}(S')$ such that $P'_i > P_i^* \forall i \in S'$. Then, using the Folk Theorem, we have that there exists $\bar{\alpha} < 1$ such that, for all $\alpha \in (\bar{\alpha}, 1)$ and for all $P^* \in \mathcal{V}_\alpha(S^*)$, there exists $S' \in \mathcal{F}$ and $P' \in \mathcal{V}_\alpha(S')$ such that $P'_i > P_i^* \forall i \in S'$ which contradicts the fact that S^* is stable.

References

- Beavis, B., Dobbs, I.: Optimization and stability theory for economic analysis. Cambridge: Cambridge University Press 1990
- Fudenberg, D., Maskin, E.: A theory of exit in duopoly. *Econometrica* **54**, 943–960 (1986)
- Ghemawat, P., Nalebuff, G.: Exit. *Rand Journal of Economics* **16**, 184–194 (1985)
- Ghemawat, P., Nalebuff, G.: The devolution of declining industries. *Quarterly Journal of Economics* **105**, 167–186 (1990)
- Greenberg, J.: Coalition structures. In: Aumann, R., Hart, S. (eds.) *Handbook of game theory*. Amsterdam: North-Holland/Elsevier Science Publishers 1994
- Friedman, J.: *Oligopoly theory*. Cambridge: Cambridge University Press 1987
- Moldovanu, B., Winter, E.: Order independent equilibria. *Games and Economic Behavior* **9**, 21–34 (1995)
- Rubinstein, A.: Perfect equilibrium in a bargaining model. *Econometrica* **50**, 97–109 (1982)
- Tirole, J.: *The theory of industrial organization*. Cambridge, Mass.: MIT Press 1988
- Wen, Q.: “Folk theorem” for repeated games with complete information. *Econometrica* **62**, 949–954 (1994)
- Wilson, R.: Strategic models of entry deterrence. In: Aumann, R., Hart, S. (eds.) *Handbook of game theory*. Amsterdam: North-Holland/Elsevier Science Publishers 1992